## Tutorial Week 5

**Definition 1.** The Dictionary Matching Automaton,  $\mathcal{D}(X)$ , it accepts the language  $A^*X$ , where X is a set of patterns, and is defined by:

- set of states is  $\operatorname{Pref}(X)$ ; initial state is the empty string;
- set of terminal states is  $\operatorname{Pref}(X) \cap A^*X$
- arcs are of the form (u, a, h(ua)), where h(ua) is the longest suffix of ua that belongs to  $\operatorname{Pref}(X)$

The trie,  $\mathcal{T}(X)$ , associated to the set X is the digital tree whose branches are labelled by strings of X (it contains only forward arcs).  $\mathcal{T}(X)$  is the basis of the Dictionary Matching Automaton,  $\mathcal{D}(X)$ .

**Exercise 1.** Consider now the following sets of patterns:  $X_1 = \{aa, abab, abaab, abba\};$  $X_2 = \{aab, abb, aaba, abab\}.$ 

For each of the sets do the following:

a) Draw their corresponding tries.

**b**) Give the failure table of each trie; what is that and how do you obtain it?

c) Draw their corresponding representation of the Dictionary Matching Automaton, including its failure links and final states.

Solution: a)





b) and c)

	j	0		1	2	•	3	4		5	,	6		7	8	3	9	
	F		. (	$q_0$	$q_1$	. q	$q_0$		L	$q_0$	)	$q_2$	9	$q_3$		1	$q_3$	
0	$Opt\_F$		-   -		$q_1$	. –	-	_	-	_	$- q_1$		9	3	$q_1$		$q_3$	
	i		0	1		2	3	2	4		5		6		7		2	
	$\frac{J}{F}$	0					-			+		_	-				,	
				q	0	$q_1$	q	0	$q_{z}$	3	q		$q_0$	q	5	q	3	
	Opt_H	<i>+'</i>	—	-	-	$q_1$	-	-	$q_{z}$	3	q	1	$q_0$	q	5	q	3	

d) The two optimised representations are the following:





**Exercise 2.** Consider now the set of patterns  $X = \{aa, abab, abaab, abba\}$ (this is  $X_1$  from the previous exercise) and the text y = abcbabaababbaaabbabaa.Fill up the following table, corresponding to the searching phase (use of the optimised failure function - note that this might generate several states for one value of j). Please underline all final states.

j		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
y[j]		a	b	a	b	a	b	a	a	b	a	b	b	a	a	a	b	b	a	b	a
state	0																				

Solution:

ſ	j		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	y[j]		a	b	a	b	a	b	a	a	b	a	b	b	a	a	a	b	b	a	b	a
	state	$q_0$	$q_1$	$q_3$	$q_4$	$\underline{q_7}, q_3$	$q_4$	$\underline{q_7}, q_3$	$q_4$	$\underline{q_6}$	$\underline{q_9}, q_3$	$q_4$	$\underline{q_7}, q_3$	$q_5$	$\underline{q_8}, q_1$	$\underline{q_2}, q_1$	$\underline{q_2}, q_1$	$q_3$	$q_5$	$\underline{q_8}, q_1$	$q_3$	$q_4$