Tutorial Week 2

Definition 1. *For a string w, an integer p with $0 < p \leq |w|$ is a* **period** *of w if for all defined positions i and $i+p$ in w, we have $w[i] = w[i+p]$ (here by $w[i]$ we refer to the symbol in position i of w, thus $i < |w|$). A string u is a* **border** *of w if u is both a prefix and a suffix of w, but $u \neq w$.*

Definition 2. *For two strings u and v, we say that u is a* **conjugate** *of v, if there exist two strings x and y such that $u = xy$ and $v = yx$.*

Exercise 1. *For the following list of strings, give the list of all their conjugates, and the full lists of their borders and their periods.*

	conjugates	*borders*	*periods*
ababab			
aaaaaa			
abcacb			
abaaba			

Proposition 2. *Two strings u and v are conjugate if and only if there exists a string z such that $uz = zv$*

Lemma 3 (Fine and Wilf – Periodicity Lemma)**.** *If a string can be written as either $u\{u,v\}^k$ and $v\{u,v\}^\ell$, respectively, for some positive integers $k, \ell > 0$, such that its length is at least $|u|+|v|$, then the string is $\gcd(|u|,|v|)$-periodic.*

Proposition 4. *If w is a primitive string, then w occurs as a factor of ww only as a prefix or as a suffix (prove using Lemma 3).*

Proposition 5. *If for strings u and v we have $u^k = v^\ell$, for some integers $k, \ell > 0$, then u and v are powers of the same string (prove using Lemma 3).*

Exercise 6. *Use the rolling hash technique to find the representation of all factors of length 5 in base 7 modulo 9, for each of the following strings:* `1234560123`*,* `2312132132`*, and* `5534555345`*. Finding only the correct value is NOT enough.*