On the construction of reversible automata for reversible languages

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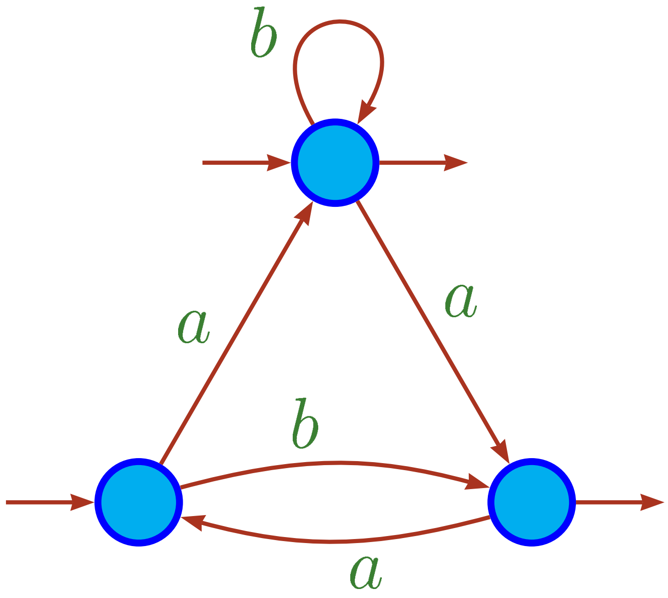
Reversible automaton

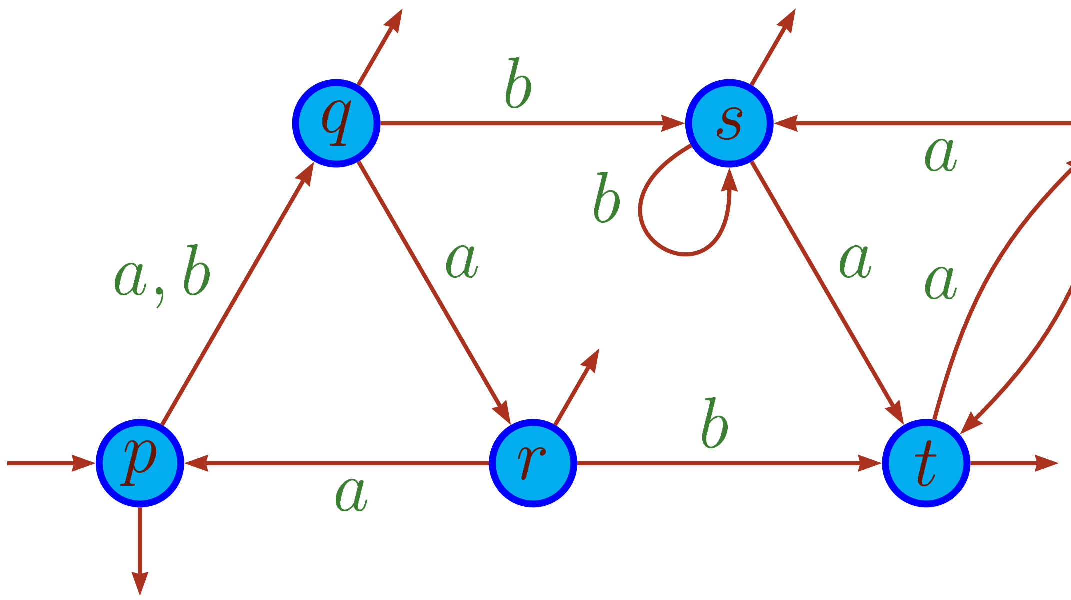
– Transition deterministic &
codeterministic automaton
– Several initial and terminal states



Reversible language

$\mathcal{L}$ reversible $\Leftrightarrow \exists\ \mathcal{A}$ reversible, $L(\mathcal{A}) = \mathcal{L}$.

Pb: $\mathcal{L}$ rev. $\not\Rightarrow$ minimal automaton of $\mathcal{L}$ is reversible



Th (Pin 92): Reversibility is decidable
in polynomial time ($O(n^4)$).

Construction of reversible automaton

Pb : How to find a reversible automaton for a given reversible language

1 Pin's algebraic characterization

$\rightarrow$ automaton with $O(mR(2,m,3))$ states.

m size of the syntactic monoid

(exponential w.r.t minimal automaton)

$R(2,m,3)$ Ramsey number

(growth unknown, computation hard)

2 Universal automaton $\rightarrow$ automaton with $O(2^{2^n})$

Universal automaton

Conway, 1971: *Regular algebra and finite machines*
$\rightarrow$ "Factor matrix"

For every regular language $\mathcal{L}$,

there exists a finite automaton $\mathcal{U}_{\mathcal{L}}$ such that :

– $\mathcal{U}_{\mathcal{L}}$ accepts $\mathcal{L}$;
– For every DFA or NFA $\mathcal{A}$ that accepts $\mathcal{L}$,
there exists a morphism $\varphi : \mathcal{A} \longrightarrow \mathcal{U}_{\mathcal{L}}$;
– $\mathcal{U}_{\mathcal{L}}$ is minimal w.r.t. this property.

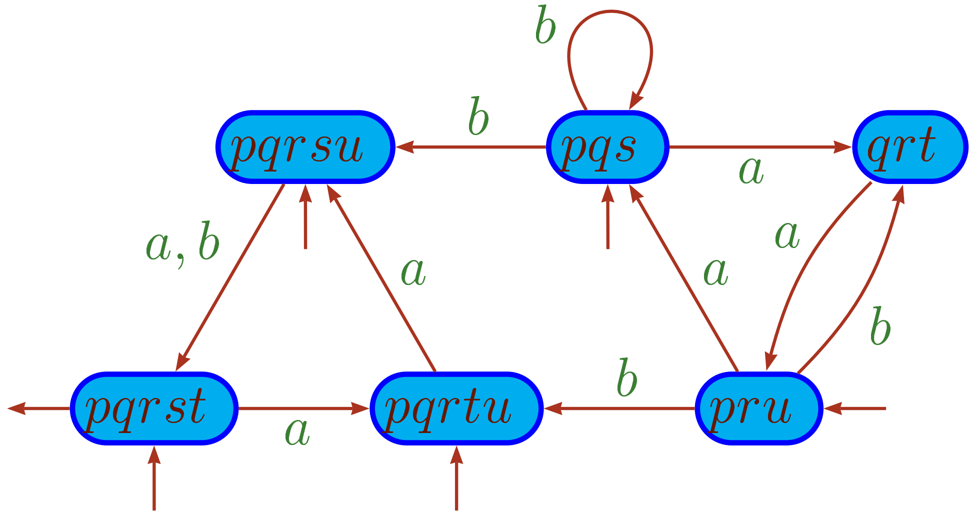
Construction of universal automaton

$\mathcal{L}$ language $\rightarrow$ $\mathcal{A}_{\mathcal{L}}$ minimal automaton
$\rightarrow$ Co-determinized automaton $\mathcal{C}$ of $\mathcal{A}_{\mathcal{L}}$
(subset construction)

Every state of $\mathcal{C}$ is a set of states of $\mathcal{A}_{\mathcal{L}}$
$\rightarrow$ compute the closure of the states of $\mathcal{C}$ under $\cap$

Th: The result is in bijection with states of $\mathcal{U}_{\mathcal{L}}$ and transitions of $\mathcal{U}_{\mathcal{L}}$ can then be read on $\mathcal{A}_{\mathcal{L}}$.

Universal automaton (example)



$$\{p,q,r,s,t\} \quad \{p,q,r,t,u\} \quad \{p,q,r,s,u\}$$

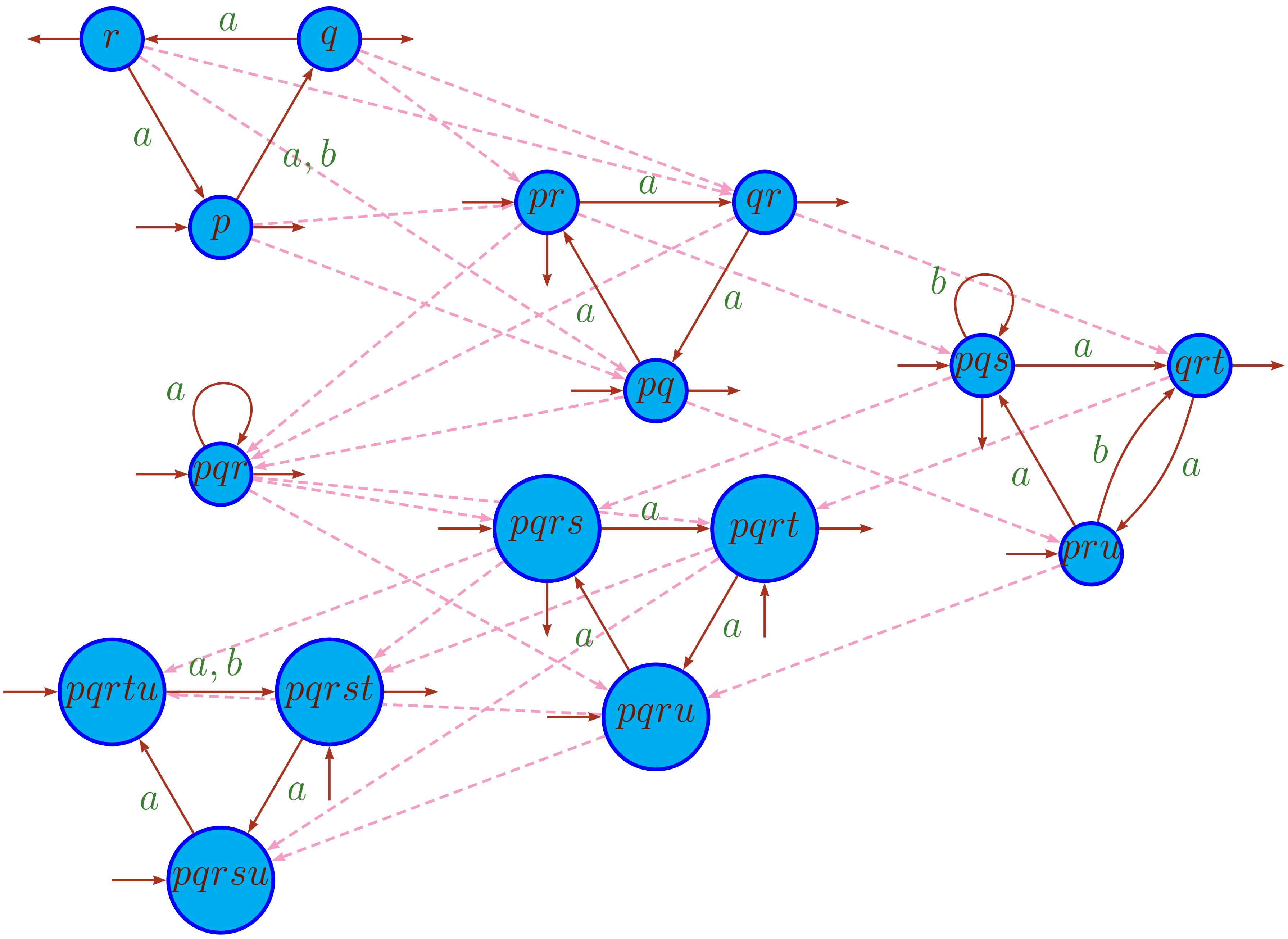
$$\{p,q,r,s\} \quad \{p,q,r,t\} \quad \{p,q,r,u\}$$

$$\{p,q,r\}$$

$$\{p,q,s\} \quad \{q,r,t\} \quad \{p,r,u\}$$

$$\{p,q\} \quad \{q,r\} \quad \{p,r\}$$

$$\{p\} \quad \{q\} \quad \{r\}$$



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Result

Th: The universal automaton of
a reversible language contains
an equivalent quasi-reversible sub-automaton.

$\Longrightarrow$ *Algorithm*

– Compute the universal automaton
– Delete transitions to obtain
the largest quasi-reversible sub-automaton
– Quasi-reversible $\rightarrow$ reversible

Rq: The size of the quasi-reversible automaton is (only) exponential w.r.t the minimal automaton.

Steps of proof

Prop: $\mathcal{L}$ reversible $\Longrightarrow$ SCC of $\mathcal{U}_{\mathcal{L}}$ reversible.

Prop: $\mathcal{A}$ minimal union of reversible SL-automata,

$$\varphi : \mathcal{A} \to \mathcal{U}_{\mathcal{L}(\mathcal{A})}$$

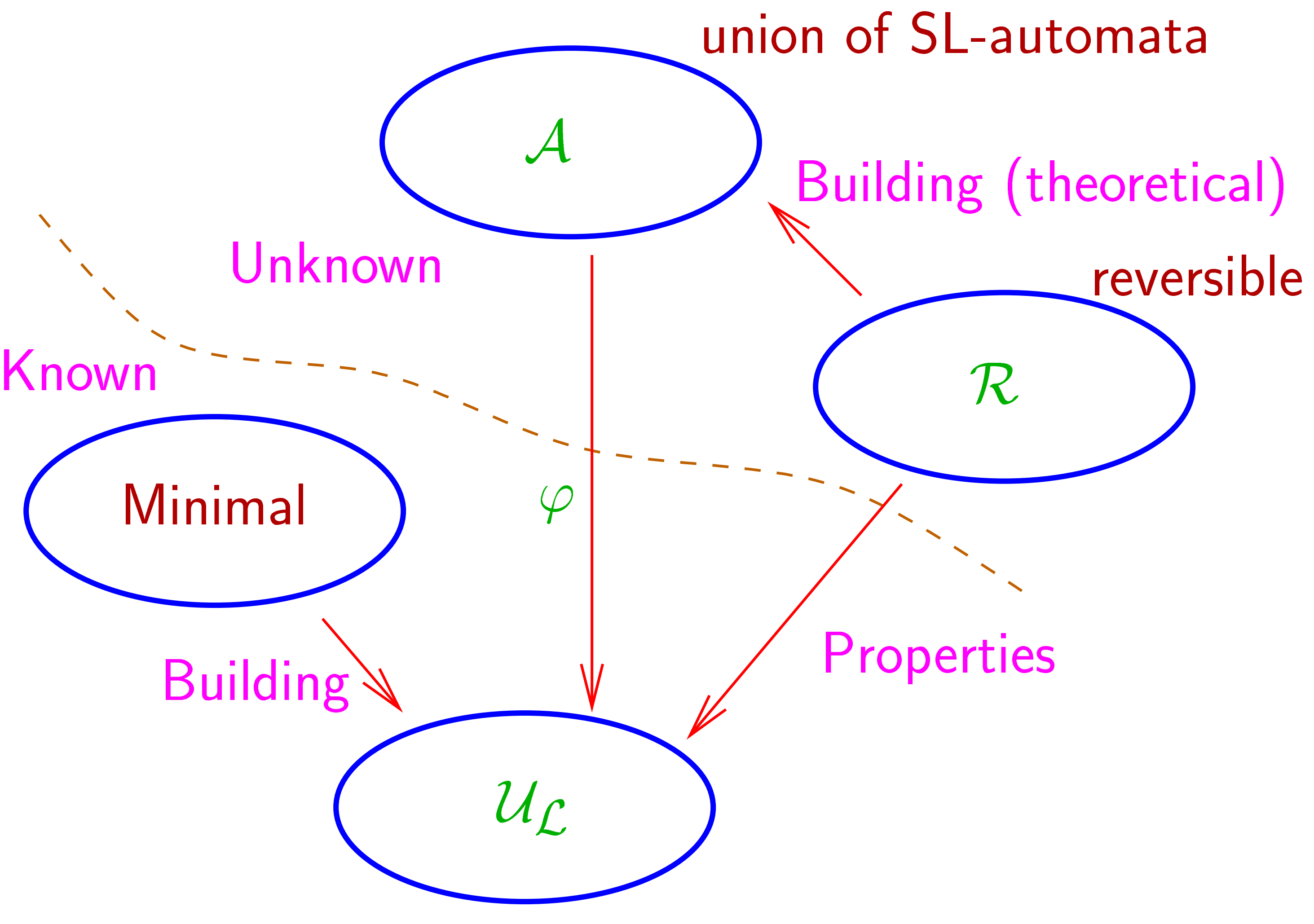
$\Longrightarrow$ $\mathcal{S}$ SCC of $\mathcal{A} \Rightarrow \varphi(\mathcal{S})$ SCC of $\mathcal{U}_{\mathcal{L}}$.

Prop: $\mathcal{A}$ minimal union of reversible SL-automata,

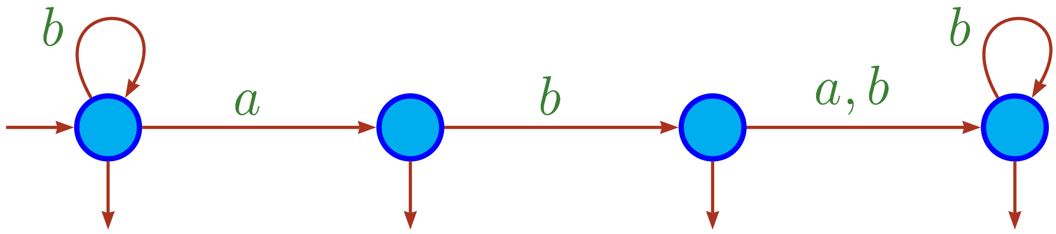
$$\varphi : \mathcal{A} \to \mathcal{U}_{\mathcal{L}(\mathcal{A})}$$

$\Longrightarrow$ $\varphi(\mathcal{A})$ quasi-reversible.

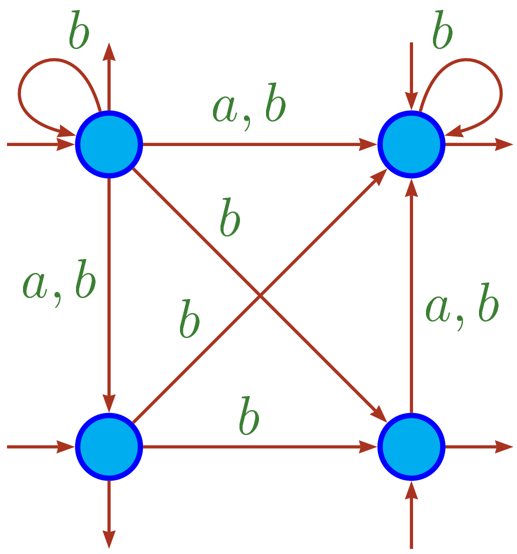
Steps of proof (2)



Example

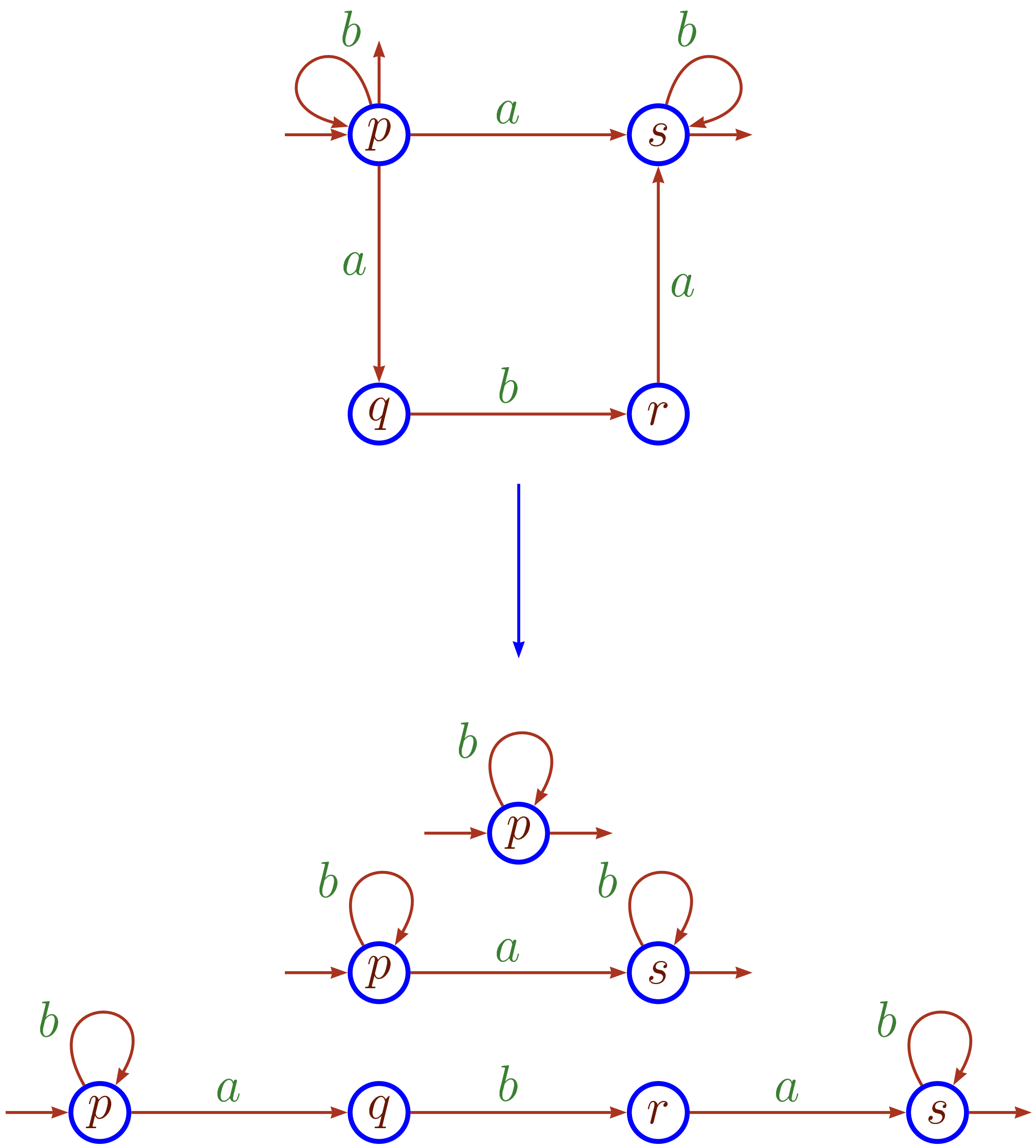


Minimal automaton

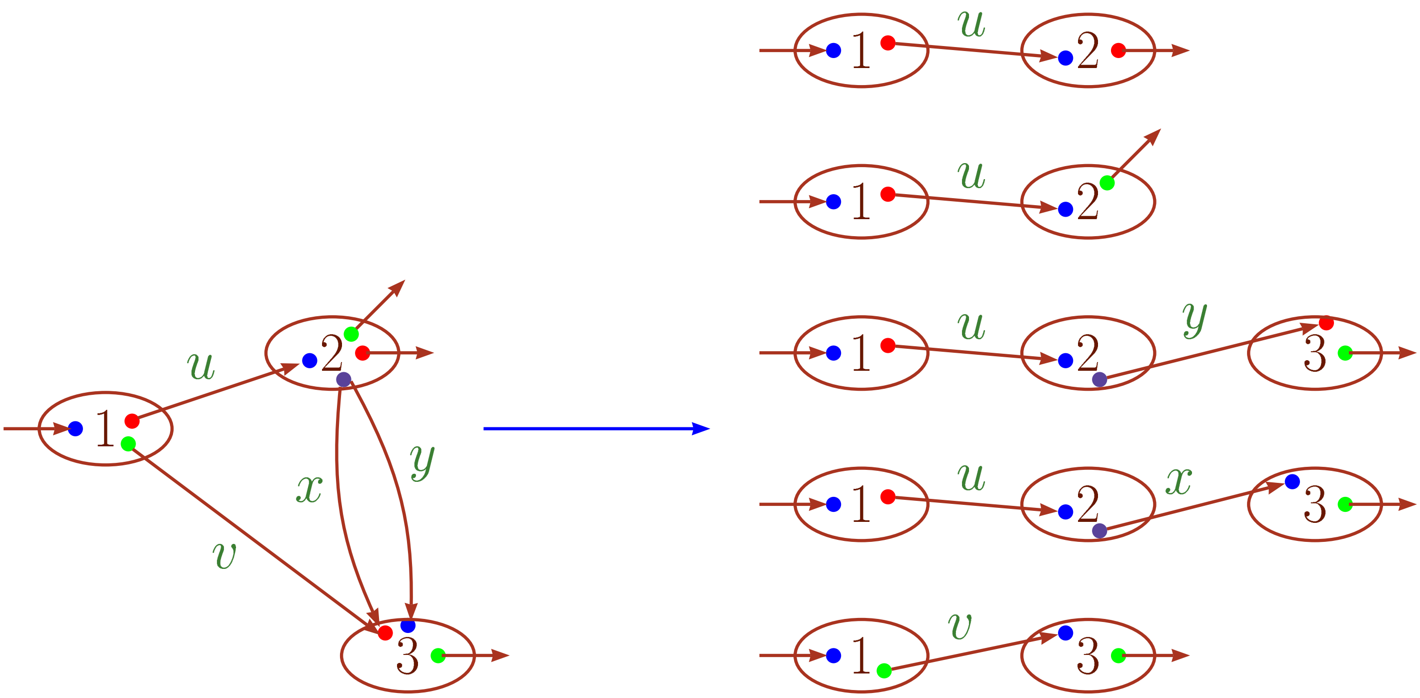


Universal automaton

Automaton $\to$ SL-automata (example)



Automaton $\rightarrow$ SL-automata



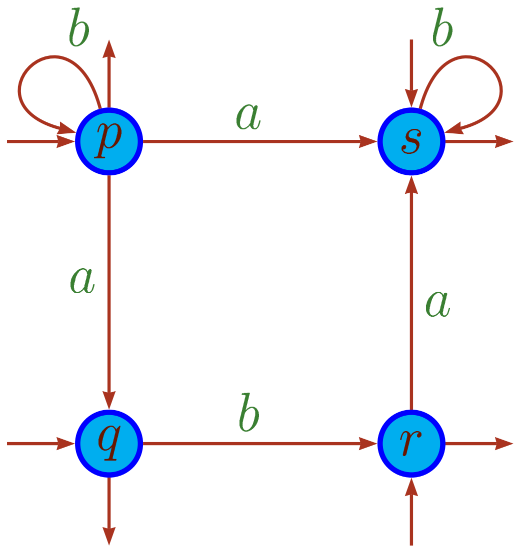
Union of SL-aut $\mathcal{A}_i$ minimal:

$$\forall i,\ \exists w \in \mathcal{L}(\mathcal{A}_i),\ \forall j \neq i,\ w \notin \mathcal{L}(\mathcal{A}_j)$$

Quasi-reversibility

If a transition foils reversibility,

it does not belong to any SCC.



A quasi-reversible automaton

Prop: $\mathcal{A}$ quasi-reversible $\Rightarrow$ $\mathcal{L}(\mathcal{A})$ reversible.

String-like automaton

– Only one initial state and one final state
– At most one transition that comes from
every strongly connected component
– At most one transition that goes to every SCC
– Trim